A perturbative approach to a nonlinear advection-diffusion equation of particle transport

Dominik Walter

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Flash Talk:

- In our contribution we analyse a nonlinear equation of particle transport
- The nonlinearity results from combining the coupled equations of particle transport and wave amplification
- The nonlineartiy manifests itself in a diffusion coefficient that is dependent on the particle distribution f
- $D = D(|f_x|)$

Resulting 1-D cartesian equation:

$$f_t + Vf_x = (D_0 \left| f_x \right|^{\nu} f_x)_x + Q_0 \qquad (1)$$

The nonlinearity is determined by the nonlinearity parameter ν. Some possible values of have already been identified e.g. by Ptuskin et al.(2009).

We apply an expansion technique, by taking:

$$f = f_0 + \nu f_1 + \nu^2 f_2 + \ldots \tag{2}$$

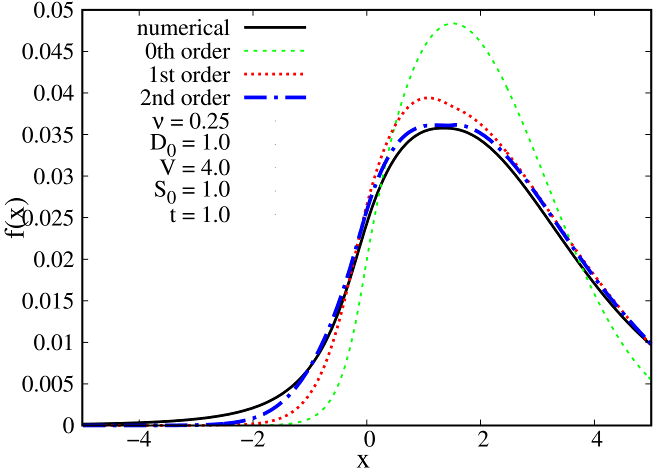
Inserting into the transport equation an equationg all terms with the same power in ν gives a set of equations.

Resulting set of equations:

$$
\begin{aligned}
\mathcal{L} f_0 &= Q_0 \\
\mathcal{L} f_1 &= Q_1 \left(f_0, x, t \right) \\
\mathcal{L} f_2 &= Q_2 \left(f_0, f_1, x, t \right) && (3) \\
\mathcal{L} f_n &= Q_n \left(f_0, ..., f_{n-1}, x, t \right) && (4)
\end{aligned}
$$

$\mathcal{L}$ is a linear operator.

- We solve this set of equations up to the second order with a numerical integraton scheme and investigate a number of different geometries of the looked at system (cartesian and spherical symmetry) and also a few different types of dependence on the nolinearity ($D = D(f_x)$, $D = (f)$, etc.).
- We investigate the quality of the derived approximations and its dependence on the nonlineariy parameter ν, the streaming velocity V, the diffusion coefficient D and the chosen source.


numerical
0th order
1st order
2nd order
ν = 0.25
D0 = 1.0
V = 4.0
S0 = 1.0
t = 1.0
f(x)
x
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
−4
−2
0
2
4