Statistical uncertainty derivation in probabilistic classification with DSEA+

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Introduction

- Values distorted through stochastic processes involved in their detection.

$$g(x) = \int A(x|y)\, f(y) dy$$

- Deconvolution: inferring the $f(y)$ distribution

- DSEA reconstructs the convoluted distribution using

$$\vec{f}_j = \sum_j^n c_{ij}$$

Introduction

- In the non-iterative case, reconstruction is given by Bayes theorem

$$n_i = \sum_j^N \frac{1}{\epsilon_i} \frac{\alpha(i|E_j) n_i^0}{\sum_m^M \alpha(m|E_j) n_m^0}$$

- In its subsequent iterations, DSEA+ replaces the prior assumption with the previous reconstruction

$$n_i^{(l)} = \sum_j^N \frac{1}{\epsilon_i} \frac{\alpha(i|E_j) n_i^{(l-1)}}{\varphi_j^{(l-1)}} = \sum_j^N U_{ij}^{(l)}$$

Error propagation

- We are searching for the covariance matrix of the solution given with

$$\Sigma_{i,m}^{(l)} = \sum^{M} \sum^{N} \frac{\partial n_i^{(l)}}{\partial \alpha(k \mid E_j)} \Sigma_{kj,on}^{\alpha} \frac{\partial n_m^{(l)}}{\partial \alpha(o \mid E_n)}.$$

- The iterative mode mandates its consideration in evaluating reconstruction's differential

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Results

- Considering the error propagation from previous iterations to subsequent reconstructions leads to an expression for the reconstruction's differential after l iterations

$$\frac{\partial n_i^{(l)}}{\partial \alpha(k|E_j)} = -U_{ij}^{(l-1)} \frac{n_k^{(l-1)}}{\varphi_j^{l-1}} + \left(\frac{n_i^{(l-1)}}{\epsilon_i \varphi_j^{l-1}}\right)_{i=k} + \sum_j^N \left[\frac{\alpha(i|E_j)\frac{\partial n_i^{(l-1)}}{\partial \alpha(k|E_j)}}{\epsilon_i \varphi_n^{(l-1)}} - \frac{\alpha(i|E_j)n_i^{(l-1)}\sum_m^M \alpha(m|E_j)\frac{\partial n_m^{(l-1)}}{\partial \alpha(k|E_j)}}{\epsilon_i (\varphi_n^{(l-1)})^2}\right]$$

non-iterative case

additional term

Results

- Narrower pull distribution of uncertainties on an unfolded toy Monte Carlo dataset

- Uncertainty depending on the number of iterations
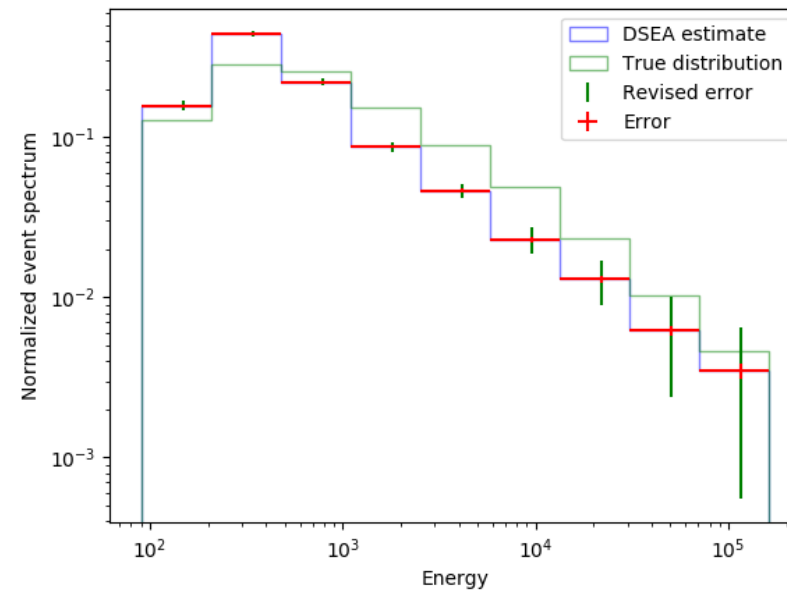
Conclusion

- The need for individually tailored inspection of error propagation depending on the working mode of DSEA+

- Improved approach to uncertainty of the reconstructed spectrum, now considering the error propagation through iteration

- Improved pull distributions of evaluated uncertainties

- Future work includes generalization of this approach to other models applicable to DSEA+

Thank you for your attention!

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for
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